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IGNITION PROBABILITY

by

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Ignition Probability

A number of studies have been made over the years in an attempt to relate pertinent weather factors (including fuel moisture) to fire occurrence. Generally, regression analyses were used. In such studies, all of the many factors that affect ignition of wildfires are necessarily included. Such studies have been useful, and in fact we will use results from them to devise an index of ignition probability, but the results do not lend themselves to improvement by further research. The only recourse is to redo the studies with different data.

The following development is an attempt to dissect ignition probability into its component parts. In this way, the place in the overall scheme of things where studies such as Blackmarr's (1969) fit, can be shown. In this way also, the places where knowledge is lacking and assumptions need to be made, can be pointed out. Hopefully, future research can then tackle specific components and make improvement of our determination of ignition probability possible.

1. Heat of preignition

The heat of preignition, defined as the net amount of heat (heat gain minus heat loss) necessary to raise the temperature of a fuel particle from its initial temperature to its ignition temperature, can be calculated by summing the following quantities:

- a. Heat required to raise the temperature of the dry fuel

from its initial temperature, T_o , to its ignition temperature T_i .

- b. Heat required to raise the moisture contained in the fuel from the initial temperature to the boiling point.
- c. Heat of desorption
- d. Heat required to vaporize the moisture.
- e. Heat required to raise the temperature of water vapor contained in the fuel voids from the boiling point to ignition temperature.

Chemical involvement of mineral components in the combustion process is neglected.

To calculate (a) for one gram of fuel, one needs to multiply the required temperature increase by the specific heat of dry fuel.

$$Q_a = (T_i - T_o) c_f \quad \text{cal/g.} \quad (1)$$

where c_f is the specific heat of dry fuel. According to Stamm (1964), the specific heat of dry wood varies with the temperature,

$$c_f = 0.266 + 0.0016t$$

where t is the average temperature between the initial temperature and 0 C. Assuming this relationship is valid for tempera-

tures up to ignition,

$$c_f = 0.266 + 0.00116(320 + T_o)/2$$

$$c_f = 0.4516 + 0.00058T_o \quad (2)$$

For T_o varying from 0°C to 70°C , c_f varies from about 0.452 to 0.492. The error in Q_a caused by using an average value of c_f would be about 2%. But the variation can easily be taken into account:

$$Q_a = (T_i - T_o)(0.4516 + 0.00058T_o) \quad (3)$$

To calculate (b), using the specific heat of water as unity, one needs to multiply the required temperature increase by the amount of water:

$$Q_b = (100 - T_o)M \quad \text{cal/g.} \quad (4)$$

where the boiling point of water is taken as 100°C and M is the fuel moisture content in fraction of dry weight.

The heat of desorption, (c), is the heat required to separate the bound water from the fibers, and is equal to the heat given off (heat of adsorption) when water vapor is adsorbed. From Stamm's (1964) figure 12-1 the following approximate equation for the heat of adsorption at any moisture content (M) may be obtained (fig. 1):

$$Q_{ad} = 280e^{-15.1M} \quad \text{cal/g.}$$

To obtain the total heat of adsorption (or desorption) we can integrate from M to $M=0$

$$Q_c = 280 \int_0^M e^{-15.1M} dM$$

$$= -18.54 \left[e^{-15.1M} \right]_0^M$$

$$Q_c = 18.54(1 - e^{-15.1M}) \quad \text{cal/g.} \quad (5)$$

The heat required to vaporize the moisture is equal to the heat of vaporization multiplied by the amount of water:

$$Q_d = 540M \quad \text{cal/g.} \quad (6)$$

If one assumes that all of the vapor leaves the system, then item (e) could be neglected. However, we would expect that the voids in the wood would be filled with vapor and that this vapor would need to be raised to the ignition temperature. We need to determine the magnitude of this heat requirement before deciding if it can be ignored.

The fractional part of the total volume of wood that is made up of voids (Stamm, 1964 p. 64) can be calculated:

$$V = 1 - g[(1/g_o) + (m_s/\rho_s) + (m/\rho)] \quad (7)$$

Since m_s and m refer to the adsorbed moisture and free moisture, respectively, the last two terms can be eliminated for our purposes. Above the boiling point there would be no free water and no bound water of consequence. Therefore,

$$V = 1 - g(1/g_o) \quad (8)$$

where V is the fractional void volume, g is the bulk dry-volume specific gravity, and g_o is the true specific gravity of wood substance.

The average value of g_o (Stamm, 1964 p. 64) is 1.46. However, g varies considerably among species and even within species. Values for common species vary from 0.3 to 0.8. Using these values and equation (8), the fractional void

volume would vary from 0.4521 for $g=0.8$ to 0.7945 for $g=0.3$.

Since our calculations of heat of preignition are being based on the mass of fuel, we will need to determine the mass of water vapor that could be contained in the voids of a unit mass of fuel. The volume occupied by one g. of wood can be obtained from the specific gravity. Using the above values for specific gravity, the volume of one g. of wood varies from 1.25 cm^3 for $g=0.8$ to 3.33 cm^3 for $g=0.3$.

The void volume of one g of wood can be calculated from the volume and the fractional void volume. For $g=0.8$

$$\text{Void volume} = 0.4521 \times 1.25 = 0.565 \text{ cm}^3$$

For $g=0.3$

$$\text{Void volume} = 0.7945 \times 3.33 = 2.65 \text{ cm}^3$$

The density of saturated water vapor at 100°C is $0.598 \times 10^{-3} \text{ g./cm}^3$ (Weast, 1965, p. E-9). Therefore the amount of water vapor (in fraction of dry weight) that can be contained in the voids is

$$M = 0.598 \times 10^{-3} \times 0.565 = .00034 \text{ g/g}$$

or 0.034% for $g=0.8$, and

$$M = 0.598 \times 10^{-3} \times 2.65 = .0016 \text{ g/g}$$

or 0.16% for $g=0.3$.

The specific heat of water vapor in the $100^\circ\text{-}320^\circ\text{C}$ range at one atmosphere pressure is approximately 0.475 (Weast, 1965, p. D-83). Therefore, the heat required to raise the temperature

of the contained water vapor from the boiling point to ignition temperature is

$$Q_e = 220 \times 0.475 \times 0.00034 = 0.0355 \text{ cal/g.}$$

for $g=0.8$, and

$$Q_e = 220 \times 0.475 \times 0.0016 = 0.67 \text{ cal/g.}$$

for $g=0.3$.

This amount of heat is negligible compared to the others involved in the heat of preignition and it can be omitted.

The heat of preignition for one gram of fuel at moisture content M then becomes

$$\begin{aligned} Q_{ig} &= Q_a + Q_b + Q_c + Q_d \text{ cal/g.} \\ Q_{ig} &= (T_i - T_o)(0.4516 + 0.00058T_o) + (100 - T_o)M \\ &+ 18.54(1 - e^{-15.1M}) + 540M \text{ cal/g.} \end{aligned} \quad (9)$$

Using $T_i = 320^\circ\text{C}$

$$\begin{aligned} Q_{ig} &= 144.51 - 0.266T_o - 0.00058T_o^2 - T_o M \\ &+ 18.54(1 - e^{-15.1M}) + 640M \end{aligned} \quad (10)$$

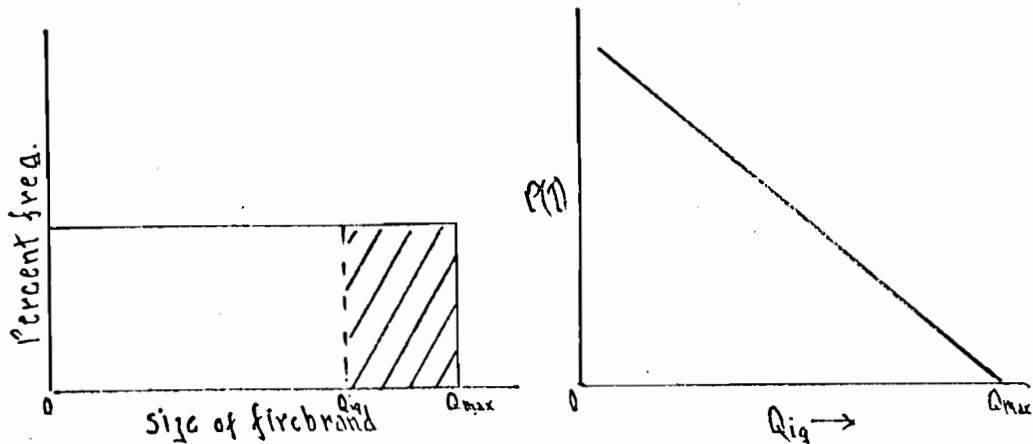
The heat of preignition calculated with equation (10) is given in figure 2.

2. Ignition probability

a. Effect of firebrand distribution.

If firebrands were evenly distributed as to size, from the smallest to the largest, the probability that a firebrand would be large enough to start a fire would be inversely proportional

to the heat of preignition (Q_{ig}). At any Q_{ig} , a firebrand providing that much heat or more would cause ignition.



This assumes, of course, that the firebrand landed on receptive fuels, and in such a way that the heat could be efficiently and rapidly transferred to the fuel.

If the firebrands in a set were all of one size, then we would expect that below a certain Q_{ig} every firebrand would start a fire, and above this critical Q_{ig} no ignition would be possible. The probability of ignition would be either 1 or 0. The same assumptions mentioned above are made.

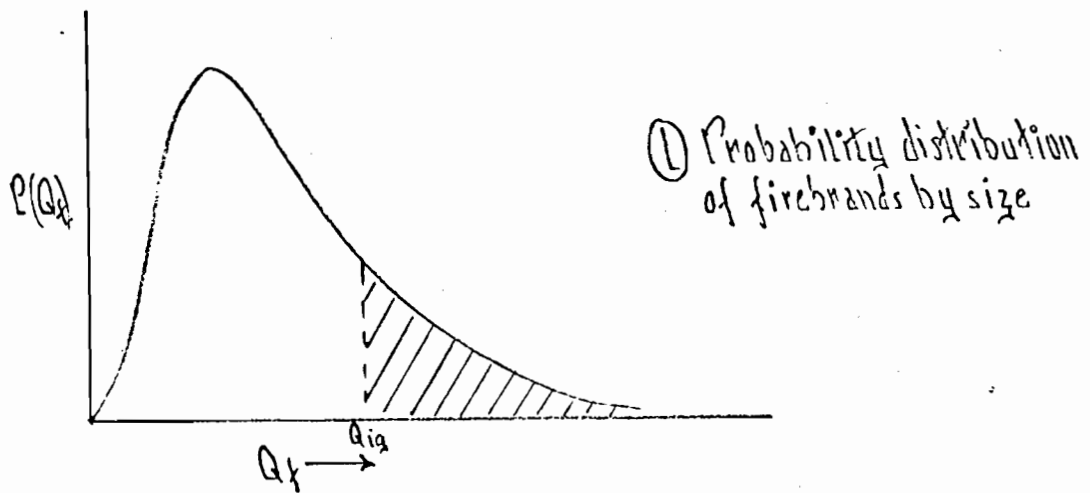
A set of larger firebrands would have a higher critical value of Q_{ig} ; and a set of smaller firebrands would have a smaller value of Q_{ig} .

It is evident that the relationship between Q_{ig} and ignition probability is dependent upon the size distribution of firebrands. In other words, ignition probability is the probability that a firebrand will produce effective heat equal to or greater than Q_{ig} .

What is the distribution of firebrand sizes? No studies have been made, and it is doubtful if such studies of firebrands in a wildland environment would be feasible.

From our experience, we would expect that, in the range of firebrands which start fires, there are more small firebrands and fewer large firebrands. One sees more sparks, for example, than larger burning embers. One would expect more flaming matches and glowing cigarette butts to be dropped on fuels in a wildland environment than, say, burning sticks or some other type of larger firebrand.

We might speculate further that the number of firebrands in the region below our range of interest would reach a peak and then drop off as the size approaches zero. In other words, the distribution would be skewed. Let us assume that the distribution of firebrands is as shown in the following illustration (1)



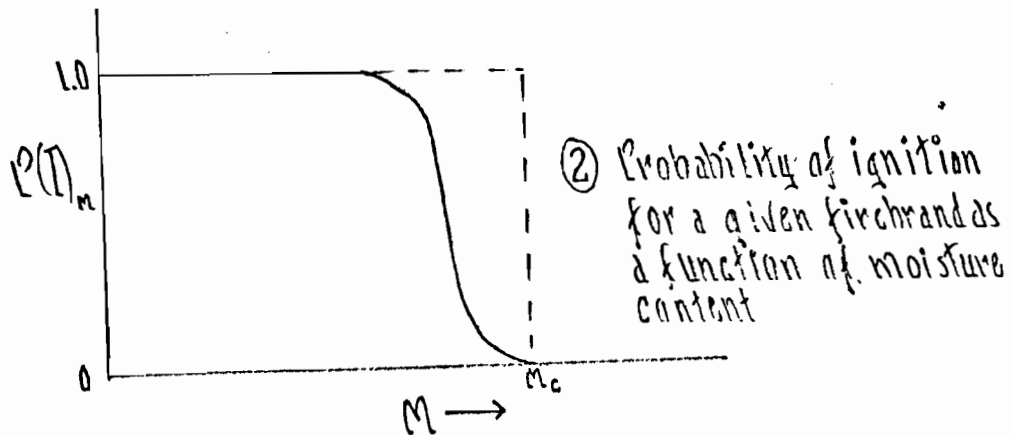
where Q_f is the effective heat of the firebrand under the most favorable heat transfer conditions. If the transfer of heat

from a firebrand, which landed on a fuel bed, to the fuel particles was always at maximum efficiency, then the probability that a firebrand would start a fire at a given Q_{ig} would equal the probability that $Q_f > Q_{ig}$. This is the area under the curve in illustration (1) from Q_{ig} to ∞ .

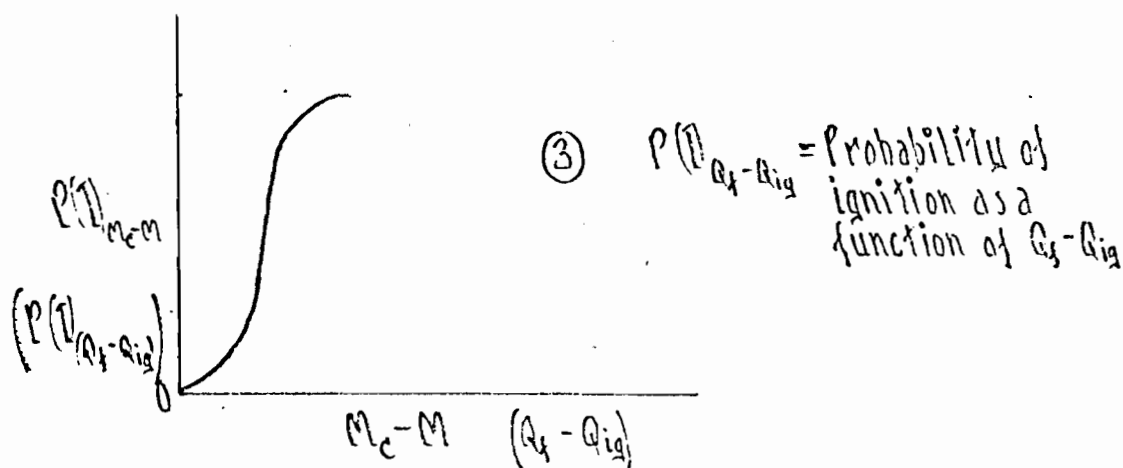
b. Effect of heat transfer efficiency

Obviously the heat transfer process is not always at maximum efficiency. Some heat is always lost and does not reach the fuel particles. Some of the heat that reaches the fuel particles is dissipated before the fuel particle reaches ignition temperature. That is, the rate of heat transfer to the fuel particles is not large enough to offset the rate of heat loss from the fuel particles.

A study by Blackmarr (1969) gives us a clue as to the efficiency of heat transfer. In this study 25 firebrands of uniform size were dropped on a fuel bed of known moisture content. The proportion of firebrands causing ignitions reflects the probability of ignition. The experiment was repeated for different fuel moisture contents and then with sets of firebrands of different sizes. The results show a reverse S-shaped curve as in illustration (2)



At maximum heat transfer efficiency for any one size of firebrand, we would expect the probability to be 1.0 below a critical moisture content and 0 above this point, as shown by the dashed line. If the critical moisture content is at the point where the curve reaches 0, then the one can relate the probability to the difference between the critical moisture content and the actual moisture content. This is the horizontal difference between the curve and the dashed line. Replotting illustration ② then gives the following

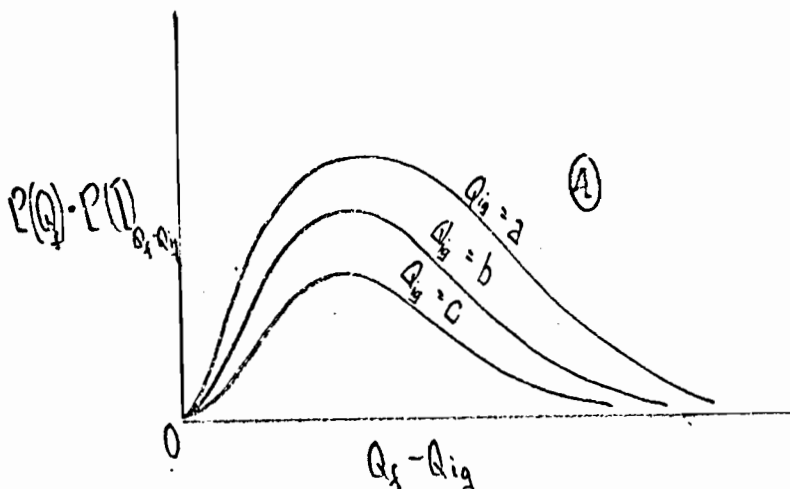


Using figure 2 we can convert moisture content to Q_{ig} if a constant temperature is assumed. (Blackmarr used constant temperature in his experiments.) (This conversion for $T=30^\circ\text{C}$ is shown in figure 5.) If the effective heat of the firebrand, Q_f , is taken to be equal to Q_{ig} at the critical moisture content, illustration ③ gives the probability of ignition as a function of Q_f-Q_{ig} . We know that no ignition is possible if Q_f is less than Q_{ig} . But if we define Q_f as the effective heat of the firebrand, it is permissible to say that $Q_f=Q_{ig}$ at the point

where $P(I)_{Q_f - Q_{ig}}$ reaches 0.

c. Obtaining ignition probability as a function of Q_{ig} .

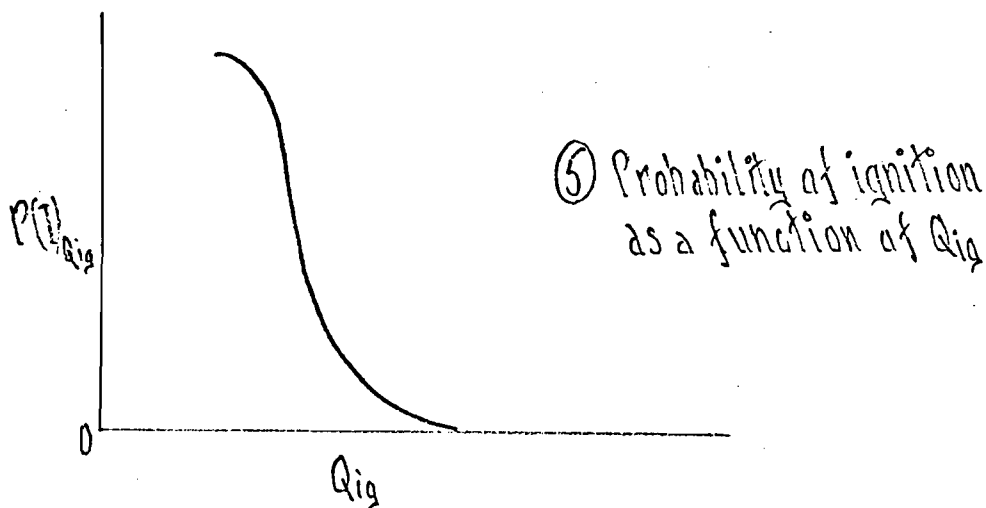
At a given Q_{ig} , we can multiply $P(Q_f)$ by $P(I)_{Q_f - Q_{ig}}$ over the range of Q_f from Q_{ig} to ∞ and obtain a family of curves as follows, (4)



The solution we want for an ignition index--that is, the probability of ignition at any value of Q_{ig} --is the area beneath the particular Q_{ig} curve.

$$P(I)_{Q_{ig}} = \int_0^{\infty} P(Q_f) \cdot P(I)_{Q_f - Q_{ig}} d(Q_f - Q_{ig})$$

If we knew the shape of this curve we could obtain the probability of ignition at any Q_{ig} . If the size distribution of firebrands curve has a shape similar to that which we assumed, the $P(I)_{Q_{ig}}$ curve would appear to be a reverse S-shaped curve, (5)



Previous studies designed to yield an ignition index or ignition probability have generally attempted to relate, through regression analyses, moisture content of fine fuels and number of fires. In such studies, the elements of ignition probability mentioned in (c) above--the distribution of firebrand sizes and the efficiency of heat transfer--are lumped together. Included also are other unspecified factors, such as fuel particle geometry, particle density, mineral content, etc., which undoubtedly affect ignition. However, since we have not yet reached the point where we can use an index with such a high degree of sophistication, we will need to consider the latter factors as having average values for the class of fuels involved in ignition.

From previous studies we can obtain a clue to the shape of the $P(I)_{Q_{ig}}$ curve. Keetch (1960) has compared the results of

several studies and several indexes based on fire occurrence and has found considerable agreement. Figure 3 is a mean curve for these studies with the highest value of ignition probability set at 1.5 percent moisture content. Since these studies were based on numbers of man-caused fires, they were concerned only with those firebrands that caused ignitions. The entire spectrum of firebrand sizes was not considered. The distribution curve, or more accurately, the $P(I)_{Q_{ig}}$ curve was truncated at some point and the highest value retained given a probability of 1.0. We do not know precisely where on the full curve the truncation point is. We can argue, however, that since all studies show ignition probability increasing at a steady or increasing rate as the lowest moisture contents are approached, the truncation point is at or to the right of the peak on the differential of the $P(I)_{Q_{ig}}$ curve.

For lack of better information, we will assume that the ignition probability for the lowest value of fuel moisture ordinarily considered is at $P(I)=0.5$ on the full curve. Making this assumption, and plotting the curve from figure 3 on log-probability paper, we find that a straight line can be fitted to the data fairly well (figure 4). In replotting, we converted moisture content to Q_{ig} at $T=30^{\circ}\text{C}$. using figure 5, which is the 30° isotherm on figure 2.

d. Backtracking to firebrand distribution

The above discussion develops the concept that the probability of ignition at any Q_{ig} is dependent upon the probability

distribution of firebrand sizes and the probability that a firebrand of a specific size will cause ignition. In fact, $P(I)_{Q_{ig}}$ is the product of the two. With $P(I)_{Q_{ig}}$ having been obtained by previous studies and shown to have a log-normal distribution, it should be possible to work backward, using information such as that obtained by Blackmarr, and determine the probability distribution of non-lightning firebrands that is inherent in $P(I)_{Q_{ig}}$ curve.

First attempts to do this have run into mathematical difficulties and the attempts will be discontinued for the present. But they should be resumed at a later date because the results would be useful. For example, the question arises as to whether one ignition index can be used for both lightning and man-caused fires. If the distribution of sizes of man-caused firebrands is similar to the distribution of lightning strikes by strength, then the use of one index is reasonable. A study by Lewis and Foust, referred to in Uman (1969) gives the distribution of 2712 cloud-to-ground lightning strikes. Plotting these data on log-normal probability paper, we can fit a straight line to them and determine the mean and standard deviations (of the log). Thus, the lightning strikes appear to have a log-normal distribution such as that shown in figure 6. We do not have, as yet, enough information on the fire-starting "long pulse" lightning strikes to account for them in this development.

e. Obtaining an ignition index

Using the above development and necessary assumptions, a reasonable ignition index can be devised.

The log-normal equation cannot be integrated except by numerical methods. However, the line in figure 4 represents an integration of the log-normal equation with $\sigma=0.139$ and $M(\log Q_{ig})=2.1761$. Since we are interested only in the truncated portion of total probability, we can devise a simple equation to fit this portion. This equation is

$$P(I)_{Q_{ig}} = \frac{.000048x^{4.3}}{50}$$

where $x = \frac{400 - Q_{ig}}{10}$

Using Q_{ig} obtained in figure 2 and the above equation, we obtain the family of curves of $P(I)_{Q_{ig}}$ in figure 7. Table 1 is a tabular representation of this set of curves. In figure 7 the probability of 0.5 has been raised to 1.0 to facilitate changing to an index number on a 0-100 scale.

In comparing this index with others that have been developed, one needs to remember that the temperature input in determining Q_{ig} is fuel temperature. To use air temperature measured at shelter height, we need to assume a temperature gradient from shelter height to the surface.

f. Obtaining an occurrence index

We will define risk as an index number related to the probable number of firebrands landing on receptive fuels per

day per some measure of area in the rating area. An objective method of determining risk should be developed by future study. For the time being, risk will need to be determined subjectively.

Defined in this way, risk can be multiplied by ignition probability to obtain an occurrence index which will be related to the probable number of fires per day in the rating area.

Table 2 has been developed in this manner.

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Table 1

NATIONAL FIRE DANGER RATING SYSTEM
 PRELIMINARY INDEX OF IGNITION PROBABILITY

FUEL TEMP.	FINE FUEL MOISTURE														OVER 30
	1.5	2.0	2.5	3.0	4.0	5.0	6.0	7-8	9-10	11-12	13-16	17-20	21-25	26-30	
30-39	87	80	74	69	59	51	43	34	25	17	10	4	1	0	0
40-49	89	83	77	71	61	53	45	36	26	18	11	5	1	0	0
50-59	92	85	79	73	63	54	47	37	27	20	11	5	2	0	0
60-69	94	88	81	76	65	56	49	39	29	21	12	6	2	0	0
70-79	97	90	84	78	68	59	51	41	30	22	13	6	2	0	0
80-89	100	93	87	81	70	61	53	42	31	23	14	7	2	1	0
90-99	100	96	90	84	73	63	55	44	33	24	15	7	3	1	0
100-109	100	99	93	86	75	66	57	46	35	26	16	8	3	1	0
110-119	100	100	96	89	78	68	59	48	36	27	17	9	3	1	0
120-129	100	100	99	93	81	71	62	51	38	29	18	9	4	1	0
130-139	100	100	100	96	84	74	65	53	40	30	20	10	4	1	0
140-149	100	100	100	99	87	77	67	55	42	32	21	11	5	2	0
150-159	100	100	100	100	90	80	70	58	45	34	22	12	5	2	0

Table 2

NATIONAL FIRE DANGER RATING SYSTEM

PRELIMINARY OCCURRENCE INDEX

		IGNITION PROBABILITY (PERCENT)																				
RISK	0	1-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99	100
10	0	3	7	12	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	100
9	0	2	6	11	15	20	24	29	33	38	42	47	51	56	60	65	69	74	78	83	87	90
8	0	2	6	10	14	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78	80
7	0	2	5	8	12	15	19	22	26	29	33	36	40	43	47	50	54	57	61	64	68	70
6	0	2	4	7	10	13	16	19	22	25	28	31	34	37	40	43	46	49	52	55	58	60
5	0	1	4	6	9	11	14	16	19	21	24	26	29	31	34	36	39	41	44	46	49	50
4	0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	40
3	0	1	2	4	5	7	8	10	11	13	14	16	17	19	20	22	23	25	26	28	29	30
2	0	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 1

Plot of figure 12-1 in Stamm

$$Q = 280e^{-15.1M} \text{ gal/g.}$$

300
200
100
90
80
70
60
50
40
30
20
10
1

0 .10 .20 .30

M

Q = gal/g. of water

KEE SEMI-LOGARITHMIC 46 5810
3 CYCLES X 140 DIVISIONS MADE IN U.S.A.
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Figure 2

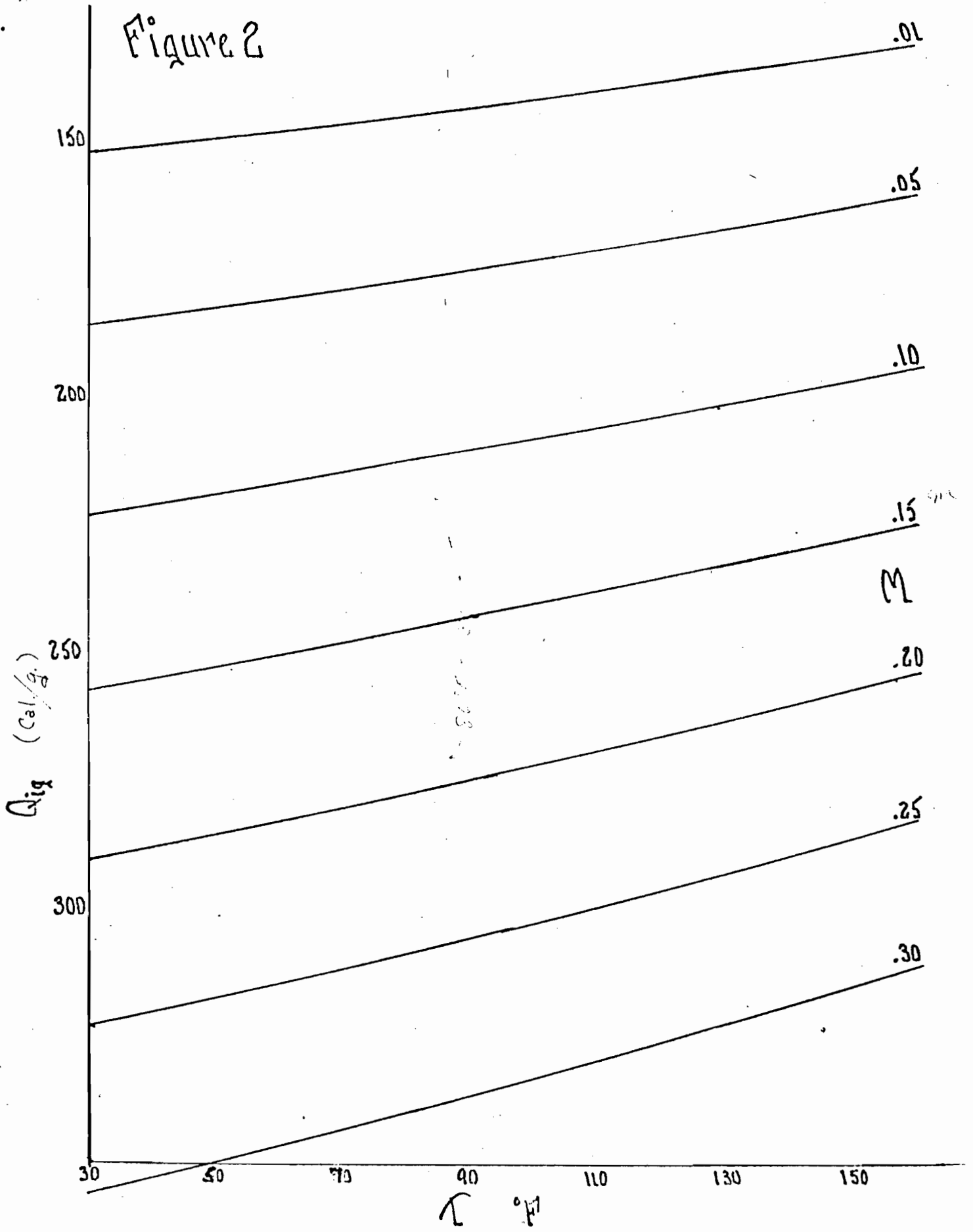
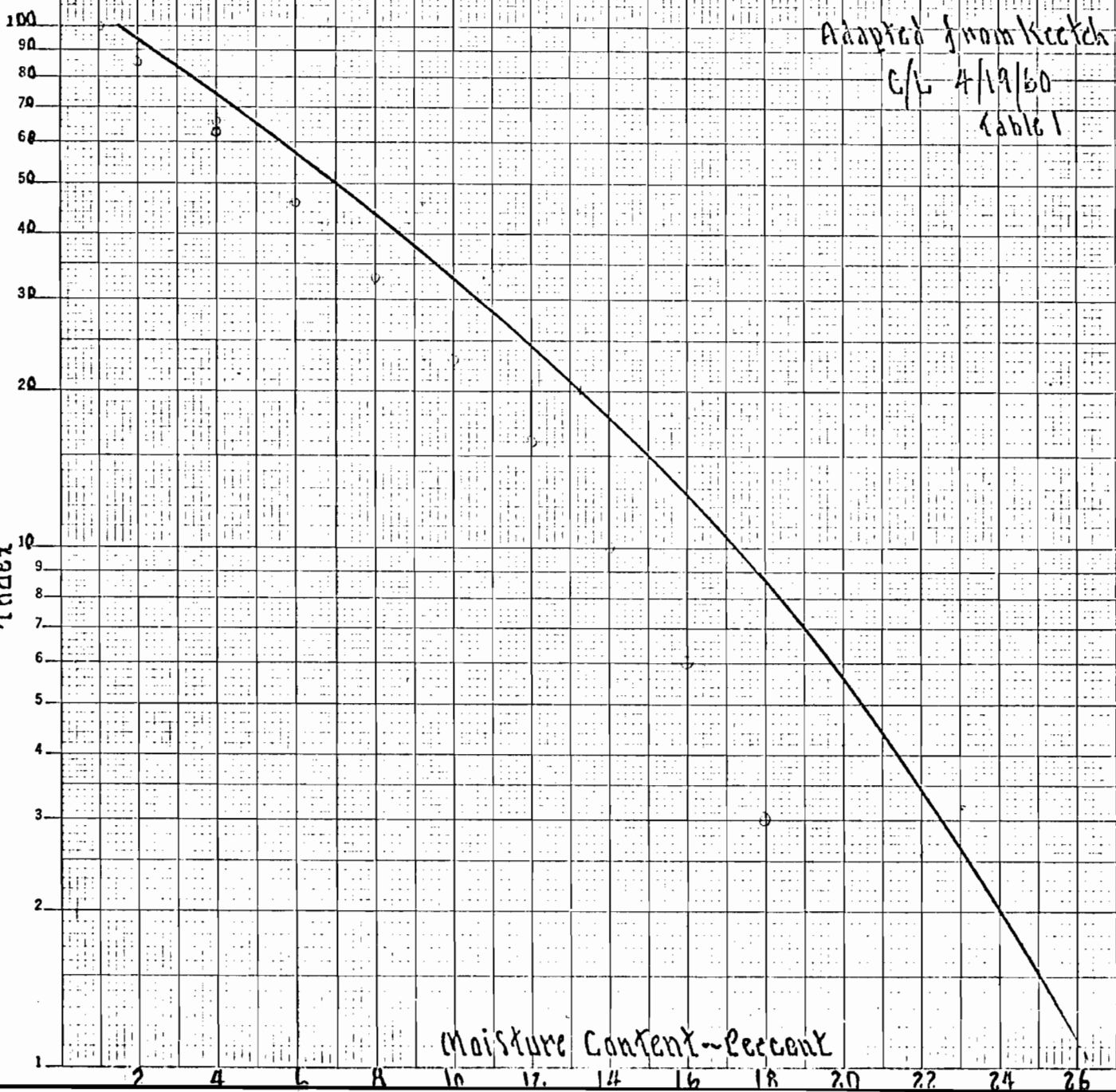


Figure 3



KE SEMI-LOGARITHMIC
3 CYCLES X 140 DIVISIONS
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Index

Moisture Content - Percent

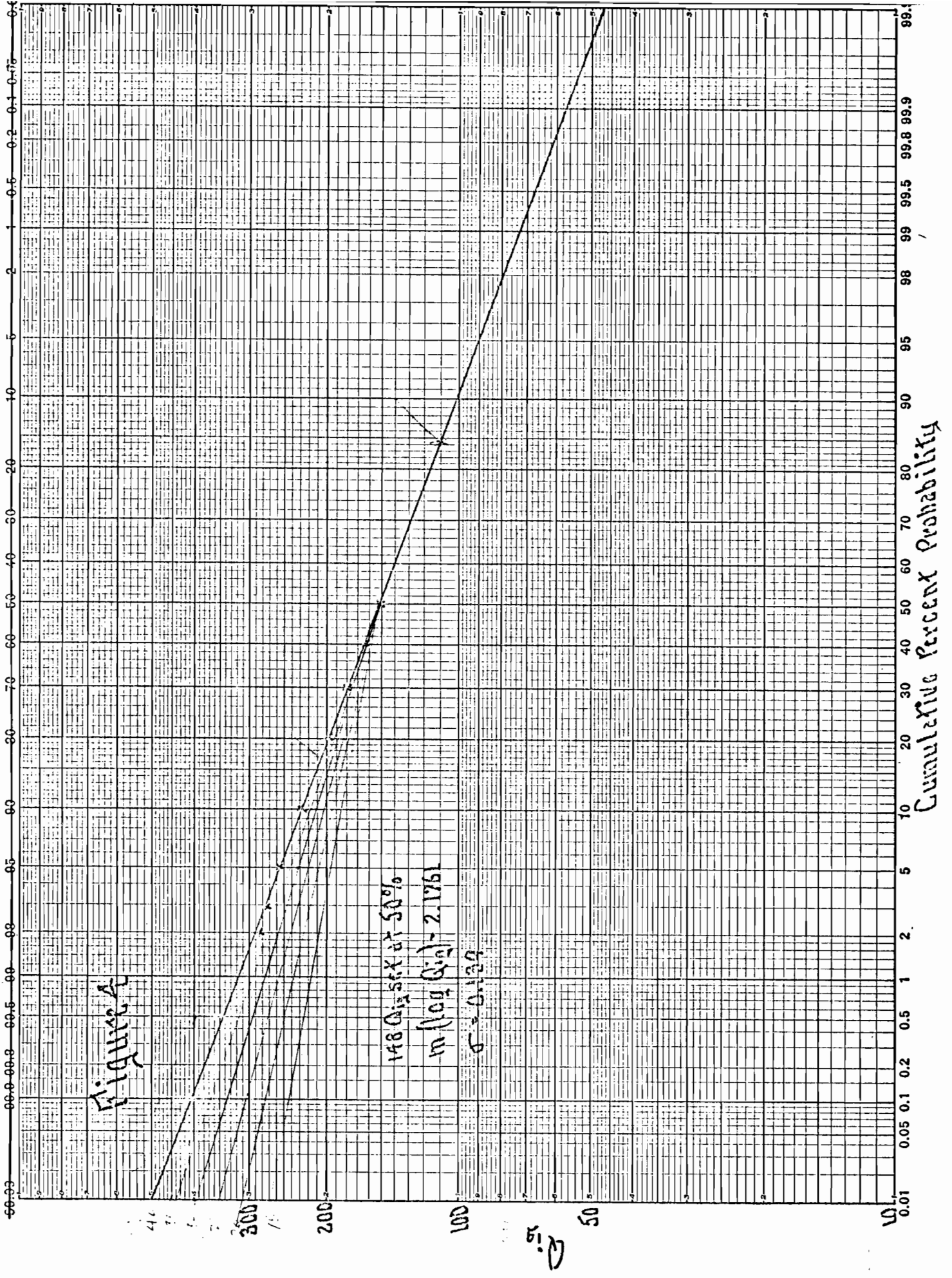
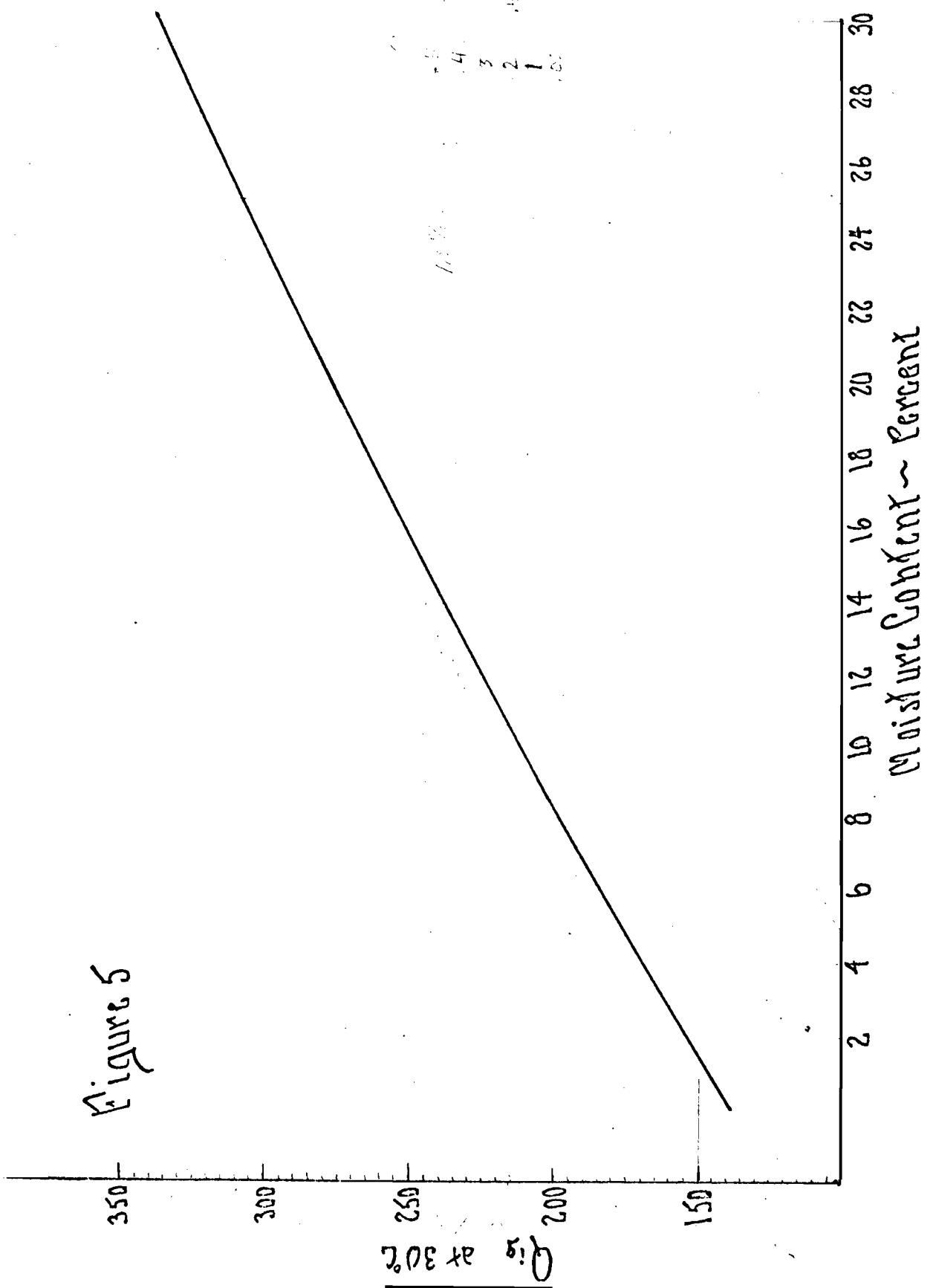


Figure 5



Qig at 30°C

Moisture Content - Percent

100
90
80
70
60
50
40
30
20
10
0

Figure 6

Distribution of peak current cloud-to-ground lightning strokes

Lewis and Moust (1945)
from Uman - "Lightning"
p. 128

Fit to log-normal curve

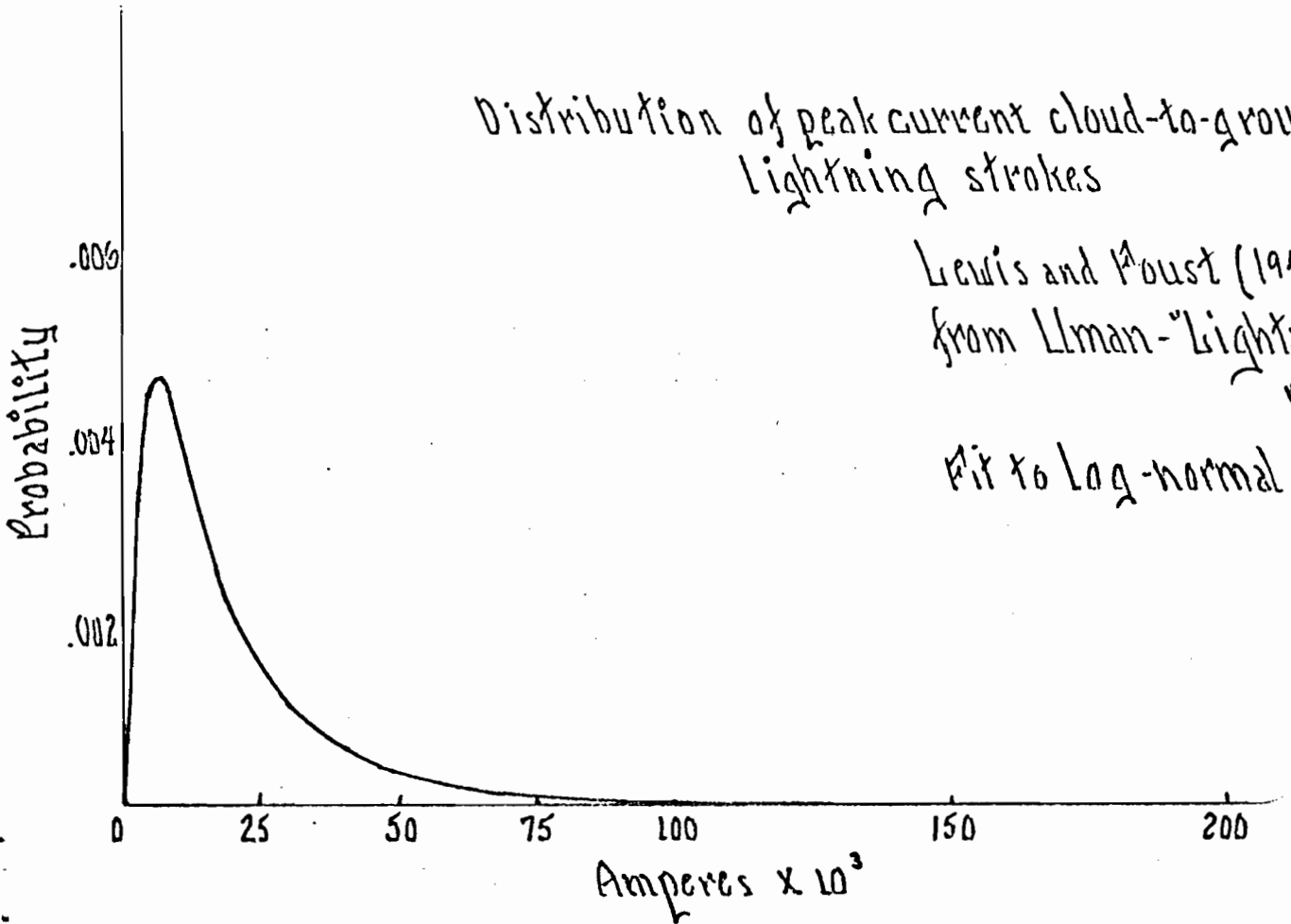
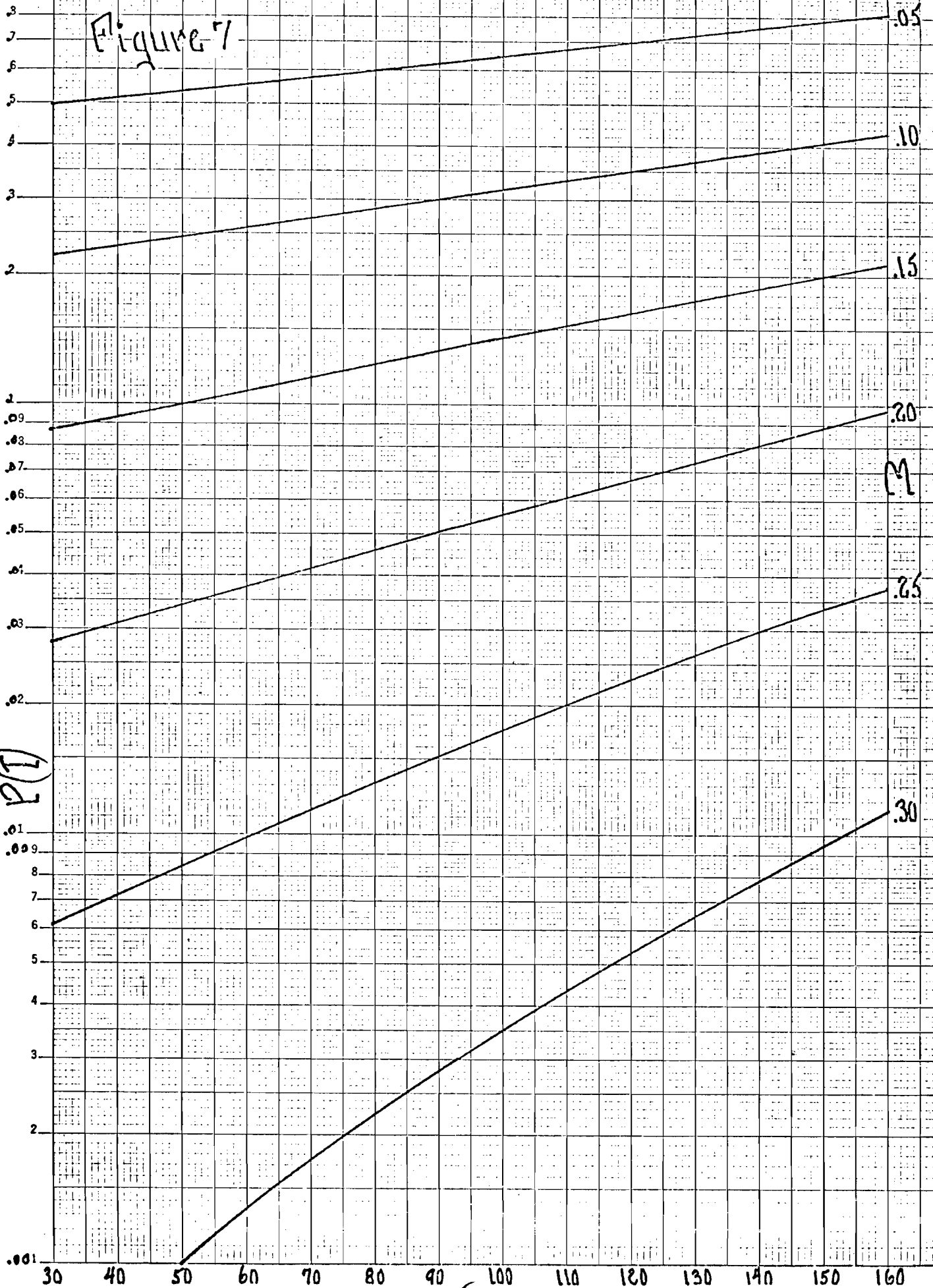


Figure 7



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P(D)

M